A Subcell Remapping Method on Staggered Polygonal Grids for Arbitrary-Lagrangian-Eulerian Methods

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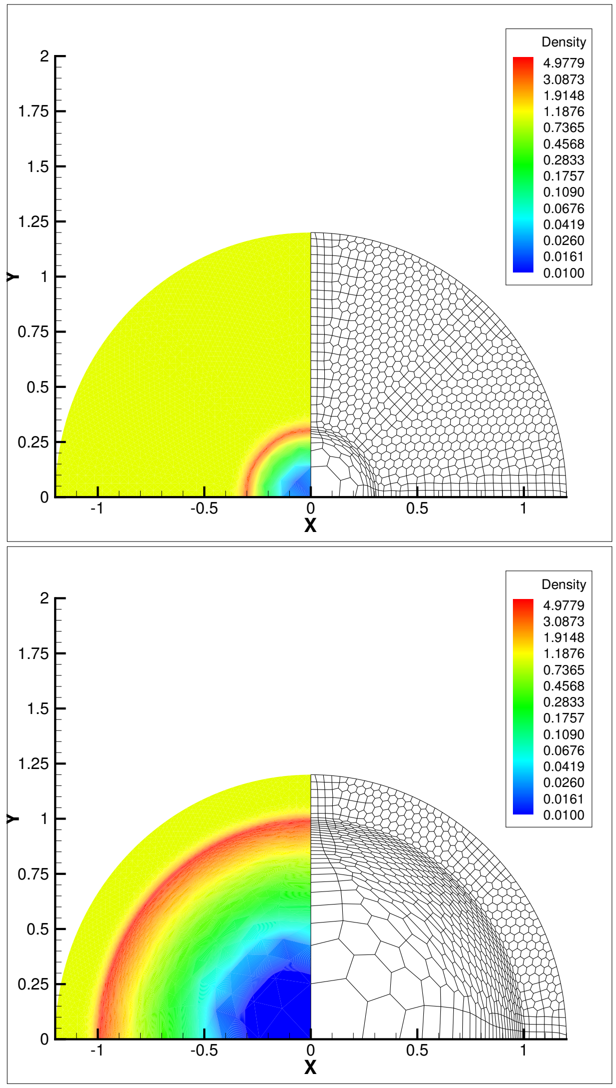
In this work we have constructed a full 2D remapping method to be used on a staggered polygonal mesh. This technique has been implemented into an ALE code. It combines and generalizes previous work on the Lagrangian and rezoning phases including this new remapping algorithm [7]. In the Lagrangian phase of the ALE method we use compatible methods to derive the discretizations [1, 2]. We assume a staggered grid where velocity is defined at the nodes, and where density and internal energy are defined at cell centers. In addition to nodal and cell-centered quantities, our discretization employs subcell masses that serve to introduce special forces that prevent artificial grid distortion (hourglass-type motions), [3]. This kind of numerical scheme adds an additional requirement to the remap phase: that the subcell densities (corresponding to subcell masses) have to be conservatively interpolated in addition to nodal velocities and cell-centered densities and internal energy. In the remap phase, we assume that the rezone algorithm produces a mesh that is "close" to the Lagrangian mesh so that a local remapping algorithm (i.e, where mass and other conserved quantities are only exchanged between neighboring cells) can be used.

Our new remapping algorithm consists of three stages.

- *A gathering stage*, where we define momentum, internal energy, and kinetic energy in the subcells in a conservative way such that the corresponding total quantities in the cell are the same as at the end of the Lagrangian



*Sedov blast wave on a polygonal mesh (*1325 *nodes and* 775 *cells) — ALE–10 regime — Mesh and density contours (exponential scale) at* $t =$ 0.1, *and* $t = 1.0$.

phase.

- *A subcell remapping stage*, where we conservatively remap mass, momentum, internal, and kinetic energy from the subcells of the Lagrangian mesh to the subcells of the new rezoned mesh.

- *A scattering stage*, where we conservatively recover the primary variables: subcell density, nodal velocity, and cell-centered specific internal energy on the new rezoned mesh.

We have proved that our new remapping algorithm is *conservative* (in mass, momentum and total energy), *reversible* (if the old and new meshes are identical then the primitve variables are kept unchanged), at least *positive* (density and specific internal energy are kept positive thanks to a repair method see [**?**]), at most, preserving a *maximum principle*, and satisfies the *DeBar consistency condition* (if a body has a uniform velocity and spatially varying density, then the remapping process should exactly reproduce a uniform velocity).
We have also demonstrated computationally that our new remapping method is robust and accurate for a series of test problems in one and two dimensions. The figure presents the results of the Sedov blastwave in 2D Cartesian coordinates for a polygonal mesh in ALE regime: a cylindrical shock wave is initiated at the origin and at $t = 1.0$ its exact location is $r = 1$. In this run the rezone strategy improves the mesh quality and the remapping technique preserves the accuracy of the Lagrangian scheme without the its pathological behaviors.

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